Kinetic roughening on rough substrates

T. J. da Silva and J. G. Moreira
Departamento de Física, Instituto de Ciências Exatas, Universidade Federal de Minas Gerais, Caixa Postal 702, 30123-970, Belo Horizonte, Minas Gerais, Brazil
(Received 14 May 1997)

We study the kinetic roughening described by a linear diffusion equation on an initially rough substrate. We show analytically that it is not possible to write a general scaling relation valid for rough substrates. However, for a particular substrate generated by the same linear growth process, we can write a scaling relation similar to the relation valid for flat substrates, with the same growth and roughness exponents. Numerical simulations in a solid-on-solid model with surface relaxation, for $d=1$, supports the scaling relation and the exponents. [S1063-651X(97)09710-9]

PACS number(s): 05.70.Ln, 68.35.Fx, 81.10.Jt

Kinetic roughening in nonequilibrium surface growth is a problem that has been extensively studied in the past decade [1–6]. Some experimental examples are the fluid-fluid interface in porous medium [7,8], paper wetting [9], propagation of flame fronts [10,11], fractures [12], molecular-beam epitaxy [4,13], and growth of bacterial cell colonies [14]. The standard theoretical methods for treating these problems are continuum equations [15,16], scaling concepts [17], and a great variety of discrete models [17,18,25,19,20].

A simplified yet nontrivial equation to describe kinetic roughening has been introduced by Kardar, Parisi, and Zhang [16]. The height $h(\mathbf{x},t)$ of the profile in position $\mathbf{x}$ at time t evolves as

$$\frac{\partial h(\mathbf{x},t)}{\partial t}=\nu\nabla^2 h(\mathbf{x},t)+\frac{\lambda}{2}[\boldsymbol{\nabla} h(\mathbf{x},t)]^2+\eta(\mathbf{x},t), \tag{1}$$

where the first term is related to the surface relaxation, the second simulates the lateral growth, and $\eta(\mathbf{x},t)$ is a white noise with zero mean and covariance given by

$$\langle \eta(\mathbf{x},t)\,\eta(\mathbf{x}',t')\rangle = 2D\,\delta^d(\mathbf{x}-\mathbf{x}')\,\delta(t-t'), \tag{2}$$

where the angular brackets denote an average over noise histories and d is the dimension of the substrate.

In 1985 Vicsek and Family [17] predicted spatial and temporal scaling properties for the interface of crystal growth. For a discrete interface of L^d sites, where the site i has a height h_i, we can define the interface width, also called roughness, as

$$w^2(t,L)=\frac{1}{L^d}\sum_{i=1}^{L^d}(h_i-\overline{h})^2, \tag{3}$$

where $\overline{h}=(1/L^d)\sum h_i$ is the mean height. For a particle deposition process in an initially flat substrate this roughness behaves as [17]

$$w(t,L)\sim L^{\alpha}f\left(\frac{t}{L^z}\right), \tag{4}$$

where α is the roughness exponent and z is the dynamical exponent. When $0\ll t\ll L^z$, the function $f(t/L^z)$ behaves as $f(t/L^z)\sim(t/L^z)^{\beta}$, where $\beta=\alpha/z$ is the growth exponent. Then the roughness has the dynamical behavior $w(t,L)\sim t^{\beta}$ for short times. For $t\gg L^z$, the function $f(t/L^z)=$ const and $w(\infty,L)\sim L^{\alpha}$. In this limit, the interface has a self-affine character.

In a growth process where the lateral growth is important, the λ term in Eq. (1) dominates. In that case, the relation $\alpha+z=2$ is valid for all dimensions and the exact solution is known only for 1+1 dimensions, with $\alpha=1/2$ and $\beta=1/3$. On the other hand, if the ν term dominates, Eq. (1) reduces to a linear equation with exact solution: $z=2$, $\alpha=(2-d)/2$, and $\beta=(2-d)/4$. All of these results are valid for a flat substrate. However, as pointed out by Meakin [3], growth from an initially rough substrate is probably more common in nature. The growth in a disordered substrate was studied theoretically by Tsai and Shapir [21], while the formation of a facet on an initially rough surface was investigated by Krug and Spohn [22].

In this Brief Report, we report analytic results of a linear equation in a rough substrate, with roughness $w_0(L)$, and we show that it is not possible to establish a general scaling relation. However, for a particular substrate, we show that $w^2(t,L)-w_0^2(L)$ obeys a scaling relation identical to Eq. (4), which is valid for flat substrates. We perform numerical simulations in a one-dimensional model of the same universality class, the solid-on-solid (SOS) model with surface relaxation [15,18]. The results obtained validate the theoretical scaling relation.

We consider Eq. (1) with $\lambda=0$, that is, the linear equation

$$\frac{\partial h(\mathbf{x},t)}{\partial t}=\nu\nabla^2 h(\mathbf{x},t)+\eta(\mathbf{x},t). \tag{5}$$

This equation has an exact solution and the roughness $w(t,L)$ can be expressed as [6,23]

$$w^2(t,L)=\int\frac{d\mathbf{q}}{(2\pi)^d}\int\frac{d\mathbf{q}'}{(2\pi)^d}C_{q,q'}(t), \tag{6}$$

where the correlation $C_{q,q'}(t)\equiv\langle h(\mathbf{q},t)h(\mathbf{q}',t)\rangle$ is given by

$$C_{q,q'}(t)=\left[e^{-2\nu q^2\Delta t}C_{q,q'}(t_0)+\frac{(2\pi)^dD}{2\nu q^2}\times(1-e^{-2\nu\Delta t q^2})\right]\delta^d(\mathbf{q}+\mathbf{q}'), \tag{7}$$

with $\Delta t=t-t_0$. For a flat substrate $C_{q,q'}(t_0)=0$ and the roughness is given by

$$w^2(t,L)=\frac{D}{2^d\nu\pi^{d/2}\Gamma\left(\frac{d}{2}\right)}\int_{\frac{2\pi}{L}}^{\infty}\frac{dq}{q^{3-d}}(1-e^{-2\nu t q^2}). \tag{8}$$

For $d=1$ this expression can be written in the scaling form

$$w^2(t,L)=\frac{DL}{\nu}f\left(\frac{\nu t}{L^2}\right), \tag{9}$$

where, for small $x\equiv\nu t/L^2$, we have $f(x)\sim x^{1/2}(w\sim t^{1/4})$ and $f(x)$ tends to a constant for large x $(w\sim L^{1/2})$. For $d=2$, the roughness has the logarithmic behaviors [23] $w(t,L)\sim\ln t$ for small t and $w(t,L)\sim\ln L$ for long times.

For a rough substrate we have $C_{q,q'}(t_0)\neq 0$ and it is impossible to obtain a general scaling relation such as Eq. (9) for any initial substrate. For example, in an uncorrelated growth process [$\nu=0$ in Eq. (5)] the correlation is given by

$$C_{q,q'}(t_0)=(2\pi)^dDt_0 \tag{10}$$

and Eq. (6) cannot be expressed as a scaling relation.

Nevertheless, if the correlation function has the form $C_{q,q'}(t_0)\sim q^{-2}$, then it is possible to write down a scaling relation similar to Eq. (9). A substrate with such a correlation function can be generated by the same growth process (5) on a flat substrate. In that case, we use a coefficient ν_0 until the time is long enough $(t\gg L^z)$. Under these conditions, Eq. (7) reads

$$C_{q,q'}(t_0)=\frac{(2\pi)^dD}{2\nu_0q^2}\delta^d(\mathbf{q}+\mathbf{q}'). \tag{11}$$

By substituting this equation into Eq. (6) we obtain

$$w_0^2(L)=\frac{D}{2^d\nu_0\pi^{d/2}\Gamma\left(\frac{d}{2}\right)}\int_{\frac{2\pi}{L}}^{\infty}\frac{dq}{q^{3-d}}. \tag{12}$$

Therefore, once the process starts on a substrate generated under those conditions, we find that the fluctuation in roughness

$$\delta w^2=|w^2(t,L)-w_0^2(L)| \tag{13}$$

can be expressed as

$$\delta w^2=\frac{D}{2^d\pi^{d/2}\Gamma\left(\frac{d}{2}\right)}\left|\frac{1}{\nu}-\frac{1}{\nu_0}\right|\int_{\frac{2\pi}{L}}^{\infty}\frac{dq}{q^{3-d}}(1-e^{-2\nu t q^2}). \tag{14}$$

This relation is identical to Eq. (8) for flat substrates with effective coefficient ν_{eff} defined as

$$\frac{1}{\nu_{eff}}=\left|\frac{1}{\nu}-\frac{1}{\nu_0}\right|. \tag{15}$$

Hence the same scaling relation is obeyed. Then, for $d=1$ the scaling relation valid for this kind of substrate is

$$\delta w^2=\frac{DL}{\nu_{eff}}f\left(\frac{\nu t}{L^2}\right). \tag{16}$$

A similar scaling relation was obtained by Kertész and Wolf for the Eden model [24]. However, the Kertész-Wolf relation depends of the intrinsic roughness of the model that is L independent, while in the present work the initial roughness $w_0(L)$ grows with L.

In order to understand the above results we perform a simulation with a model of the same universality class, the so-called SOS model with surface relaxation, in $d=1$. That model was introduced by Edwards and Wilkinson [15], who determined the exponents analitically. Family [18] obtained numerical results that support the scaling relation (4) with the same exponents. A site i is select at random and its height h_i grows one unit, that is, $h_i\rightarrow h_i+1$, provided that the restriction on neighboring heights $h_i-h_{i\pm1}<m$ is obeyed at all stages. In this constraint, m is a parameter related to the coefficient ν. In case the constraint is violated, the lowest neighboring site is visited until a local minimum is reached. A substrate is generated by this process by using a value m_0 until a steady state $(t\gg L^z)$ is attained. On that

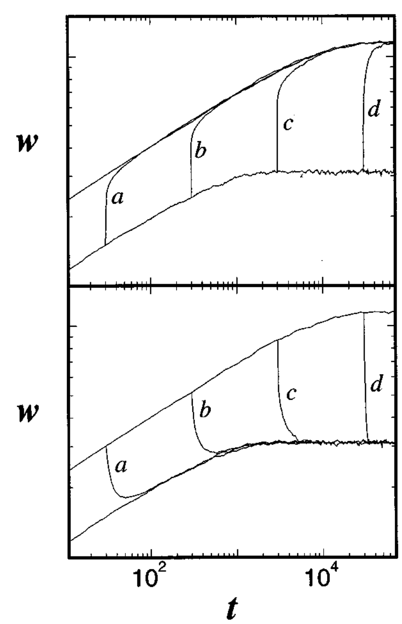

FIG. 1. The log-log plots of the roughness $w(t,L)$ as a function of t for $L=200$. In both parts, we show the growth process in a flat substrate for $m=1$ (lower curve) and for $m=5$ (upper curve). On the top roughening processes from $m=1$ to $m=5$ and on the bottom smoothening processes from $m=5$ to $m=1$ are described. The segments a, b, c, and d show the behavior when we change the parameter m at times $t=30$, 300, 3000, and 30 000, respectively.

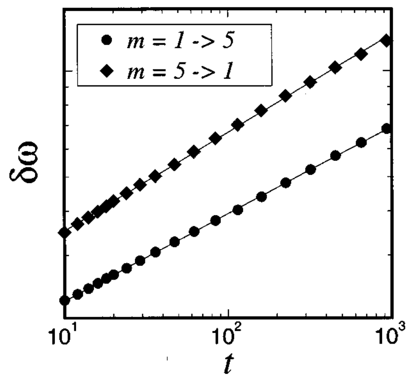

FIG. 2. The log-log plots of the fluctuation in roughness δw as a function of t in two process: (a) roughening (lower curve) (in an $L=800$ less rough substrate, generated with $m=1$, grows a structure with $m=5$) and (b) smoothening (upper curve) (in an $L=800$ very rough substrate, generated with $m=5$, grows an interface with $m=1$.

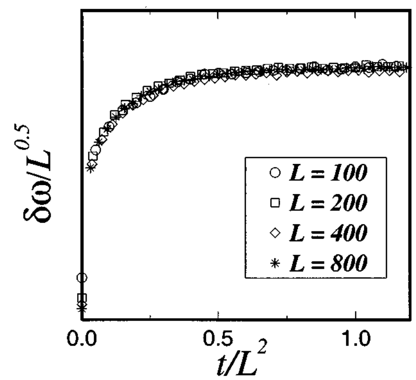

FIG. 3. Plots of $\delta w/L^{1/2}$ vs t/L^2 for various values of L showing the validation of the scaling relation (16).

substrate another growth process will begin with another maximum height difference $m \neq m_0$. The change in the parameter m corresponds to a change in the coefficient ν. For instance, a low value of m means a less rough substrate, that is, a higher value of the coefficient ν.

Figure 1 shows log-log plots of the roughness $w(t,L)$ vs time t for $L=200$. The lower (upper) curves are for $m=1$ ($m=5$). The segments a, b, c, and d on the top describe the roughening from $m=1$ to $m=5$ at different times. The segments on the bottom describe the smoothening processes $m=5$ to $m=1$. We note that there exists a tendency of the roughness to go quickly to the corresponding curve of the new value of m. Even for the smoothening process the segment a shows a strong decreasing of the roughness and then it returns to the ascending curve.

Figure 2 shows the absolute value of the fluctuations of roughness δw vs time t for two growth process: (a) roughening, a process with $m=5$ on a less rough substrate generated with $m_0=1$, and (b) smoothening, a process with $m=1$ on a very rough substrate generated with $m_0=5$. For each one, we have generated 30 substrates and 30 growth processes for each substrate. Hence we used effectively 900 samples. The results shown are for $L=800$. The curves are in good agreement with $2\beta=0.48\pm0.02$ for roughening and $2\beta=0.51\pm0.03$ for smoothening, which indicates $\beta\approx 1/4$, as expected. The validation of the scaling relation (16) can be inferred from Fig. 3: Plots of $\delta w/L^{1/2}$ vs t/L^2, for several values of L, collapse on the same curve, which indicates that $f(x)$ is L independent.

In conclusion, we analyze the kinetic roughening on an initially rough substrate. Through a solution of a linear diffusion equation we show that it is impossible to obtain a general scaling relation as such Eq. (9), which is valid for flat substrates. Nevertheless, for a particular substrate generated by the same linear process but with a different coefficient ν, we can write down a similar scaling relation (16) which is a generalization of Eq. (9). It is possible to study analytically other models that are related to a linear equation of higher order [19,20] and to obtain similar results. Numerical simulations in a SOS model with surface relaxation, where a difference m of height between neighbors is allowed, indicate the correctness of this scaling relation. We also perform numerical simulations in a SOS model with refuse [25] to see the results of a nonlinear model on a rough substrate.

We thank João Florêncio, Jr. and Bismarck Vaz da Costa for helpful criticism of the manuscript. This research was supported in part by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and in part by Fundação de Amparo à Pesquisa do Estado de Minas Gerais (Fapemig), Brazilian agencies.

[1] *Dynamics of Fractal Surfaces*, edited by F. Family and T. Vicsek (World Scientific, Singapore, 1991).
[2] T. Vicsek, *Fractal Growth Phenomena* (World Scientific, Singapore, 1992).
[3] P. Meakin, Phys. Rep. **235**, 189 (1993).
[4] A.-L. Barabási and H. E. Stanley, *Fractal Concepts in Surface Growth* (Cambridge University Press, Cambridge, 1995).
[5] T. J. Halpin-Healey and Y. C. Zhang, Phys. Rep. **254**, 215 (1995).
[6] J. Krug, Forschungszentrum, Julich, Report No. JVL-3031, 1995 (unpublished).
[7] M. A. Rubio, C. A. Edwards, A. Dougherty and J. P. Gollub, Phys. Rev. Lett. **63**, 1685 (1989).
[8] S. He, G. L. M. K. S. Kahanda, and P.-Z. Wong, Phys. Rev. Lett. **69**, 3731 (1992).
[9] L. A. N. Amaral, A.-L. Barabási, S. V. Buldyrev, S. Havlin, and H. E. Stanley, Phys. Rev. Lett. **69**, 3731 (1992).
[10] A. Pocheau, Europhys. Lett. **20**, 401 (1992).
[11] J. Zhang, Y.-C. Zhang, P. Alstrøm, and M. T. Levinsen, Physica A **189**, 383 (1992).
[12] *Disorder and Fracture*, edited by J. C. Charmet, S. Rous, and E. Guyon (Plenum, New York, 1990).
[13] S. Das Sarma, C. J. Lanczycki, R. Kotliar, and S. V. Ghiasas, Phys. Rev. E **53**, 359 (1996).

[14] T. Vicsek, M. Cserzö, and V. K. Horváth, Physica A **167**, 315 (1990).
[15] S. F. Edwards and D. R. Wilkinson, Proc. R. Soc. London, Ser. A **381**, 17 (1982).
[16] M. Kardar, G. Parisi, and Y.-C. Zhang, Phys. Rev. Lett. **56**, 889 (1986).
[17] F. Family and T. Vicsek, J. Phys. A **18**, L75 (1985).
[18] F. Family, J. Phys. A **19**, L441 (1986).
[19] D. E. Wolf and J. Villain, Europhys. Lett. **13**, 389 (1990).
[20] S. Das Sarma and S. V. Ghaisas, Phys. Rev. Lett. **69**, 3762 (1992).
[21] Y.-C. Tsai and Y. Shapir, Phys. Rev. Lett. **69**, 1773 (1992).
[22] J. Krug and H. Spohn, Europhys. Lett. **8**, 219 (1989).
[23] T. Nattermann and L.-H. Tang, Phys. Rev. A **45**, 7156 (1992).
[24] J. Kertész and D. Wolf, J. Phys. A **21**, 747 (1988).
[25] J. M. Kim and J. M. Kosterlitz, Phys. Rev. Lett. **62**, 2289 (1989).